Replicator dynamics in value chains: explaining some puzzles of market selection*

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Abstract

The pure stylized selection model of replicator dynamics proposed by Metcalfe (1994) though providing important insights in the evolution of markets has not found much of empirical support (Cantner, 2014). This paper extends the model to the case of firms vertically integrated in value chains. Cost of switching one's supplier are high while fitness of partners along the value chain may differ considerably. Using a set of analytical and computational exercises we show that i) this contributes not just to a reduction of market share reallocation dynamics, but may revert its effect so that firms with low fitness gain in market share because of being integrated with partners with high fitness and the other way around; ii) relaxing the assumption of constant routines (no innovation), one can also find that fitness of partners along the value chain affects one's own fitness offering a new explanation on why some initially weak firms can quickly catch-up in terms of performance while strong firms may start lagging behind; iii) allowing partner's switching within a value chain illustrates that periods of instability in the early stage of industry life-cycle may come from 'optimization' of partners within a value chain rather than decreasing returns to scale (Mazzucato, 1998); iv) there are distinct differences between layers of a value chain causing strategic advantages to firms in partnering. Furthermore, the model replicates some empirical stylized facts on firm size and degree distribution among firms integrated in 'value chain network'.

Keywords: *innovation, network, replicator dynamics, value chain.*

JEL Classification: C63, D24, L14, O32.

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1 Introduction

This paper studies the role of integration into value chains on competitive market selection. Under the integration into value chains we mean that the performance of firms is not simply dependent on its own fitness (e.g., productivity) but also on the fitness of its partners with which it is vertically integrated to produce a finished good for consumers. Thus, in contrast to the original stylized model of market selection dynamics developed by Metcalfe (1994) (also known as 'replicator dynamics model'), we argue that in reality firms are (vertically) integrated into complex value chains adding value to a specific material/semi-finished good before it will be sold to its end consumers. This is confirmed not only by marketing research reporting that the value of business-to-business (B2B) contracts in many industries is exceeding the one from the business-to-consumer (B2C),[1] but also by numerous studies pointing to the fact that in the modern economy the degree of specialization is constantly increasing and instead of conducting the entire production cycle in-house, many stages get outsourced to firms specializing in certain area (exhibiting higher productivity in performing those stages of production).[2] An important feature of that vertical integration, however, is that firms collaborating on a long-term basis adjust their production process to one each other so that switching one's partner becomes a very (if not prohibitively) costly issue. As a result, over time a firm may get locked into cooperation with less fit partner, which has a direct impact on the firm's performance, and consequently, on the industrial dynamics of the market, where this firm is operating.

The principle of reallocation of market shares from less efficient firms to their more fit competitors is the key principle of selection-based theories (Friedman, 1953), which also play an important role in the evolutionary economics literature (Nelson and Winter, 1982). However, when it comes to empirical testing of the theory, evidence of that principle is at best mixed and at worst contradictory. Thus, evidence from industrialized countries based on productivity decompositions, where aggregate productivity growth is decomposed into i) firm-specific changes in productivity levels (fitness of a firm directly contributing to the productivity of an industry), ii) changes due to reallocation of market shares between firms (i.e. better fitness allowing to gain a larger market share counts more on the aggregate level) and iii) contribution of new firms entering the market and existing leaving it, consistently report the firm-specific source dominating the one from market reallocation, while few indicate that the contribution of market share reallocation to productivity is negative[3] (see Isaksson (2009) for an overview of **cross-country comparison but not cross-industry one**).

Hence, there must be a mechanism explaining the inconsistency described above. One explanation for this being often referred in literature is the fact that firms compete in most cases not over homogenous but heteroge

neous products (horizontal product differentiation), which reduces pressure from the side of price competition (in other words, competition in efficiency) and introduces other

[1] The major reason for this is that in a typical supply value chain (VC) there will be many B2B transactions involving sub-components or raw materials, and only one B2C transaction, namely sale of the finished good to the end customer. For example, a computer manufacturer makes several B2B transactions such as buying microchips, different cables, cooler, which producers in their turn buy nanometer transistors, rubber, plastic and metal.

[2] Speaking in words of Alexander and Young (1996): 'if you keep everything in-house, you will never generate as much'.

[3] In other words, firms, which market share was expected to increase due to its (relatively good) productivity, was in fact falling.

factors affecting firm performance (including marketing strategy). Another rationale comes from the side of mobility barriers, where as Hölzl (2015) has recently demonstrated, the speed of selection is reduced in case firms face sunk costs of exiting a market. However, these reasons, though well argued, do not explain the negative effect of productivity on market shares observed empirically, which is the first motivation for the current study. The second comes from the fact that firms in fact are deeply integrated in complex value chains in producing finished goods and by integrating this important aspect into the original replicator dynamics model, we aim to explore the resulting implications for industrial dynamics.

The paper proceeds as follows. The next section provides a literature review together with hypotheses to be tested. Section 3 describes the model of the replicator dynamics into the value chain context while in Section 4 its main results are summarized. Section 5 discusses policy implications and concludes.

2 Literature review and hypotheses

This section is work in progress...

3 Model

First of all, we have to clarify the notation we use in this study on value chain analysis. As production process of a final good includes several stages conducted by different firms (located on different markets, e.g. market of resources, market of intermediate goods and finally market of final goods), the primary unit of interest and firm performance measure is not unit cost of production (as in Metcalfe (1994)), but *unit cost per value added* (c_m^i of firm i on market m). Thus, for simplicity we assume that we can measure how much of resources and, later on, intermediate goods are necessary to produce one unit of final good, and concentrate on cost a firm has to bear to transform a certain input into respective output necessary to produce later one unit of final good.

Assumptions:

- there are M (let us start for simplicity with three) vertically integrated markets, where on each market N_m firms are operating. One can refer to those three layers as 'suppliers', 'manufacturers' and 'distributors';

- no firm can produce a finished good alone, but only in cooperation with firms in other markets. Thus, we leave out the possibility of vertical integration with one single firm present on more than one layer (market);

- we abstract ourselves from entry and exit behavior to isolate the effect of selection dynamics ($\forall m = 1, ..., M \ N_m = \text{constant}$);

- for the sake of simplicity, we also ignore sources of uncertainty for value chains such as demand (volume and product specification), process (e.g., machine downtime and transportation reliability) and supply (e.g., delivery reliability) described in detail in Strader et al. (1998). Instead, we assume perfect collection and sharing of information between supply chain members, which results in no inventories necessary and order fulfillment cycle time being minimized. Such a perfect management of lead-time in turn presents a barrier for supply chain members to switch their partners since tuning of this management is costly in terms of time and resources.

- all firms have constant returns to scale, unless specified differently;
- also for simplicity, goods on all markets (including the market of finished good M) are homogenous;
- firms on all M layers seek to earn profit. Thus, on all M layers profit margin per unit of output firms charge is fixed (parameter $\phi = 0.1 = \frac{p^i_m - c^i_m}{c^i_m}$ (10%)). In principle, one could abandon that parameter. However, since firms in our model conduct cost-reducing R&D, we add positive profits to add realism to the model. In addition, one can later investigate the role of that parameter for the output produced by the model;
- another standard assumption from replicator model adopted in this study is the investment in capacity extension: whenever a firm makes profit by selling its output at a price above its costs, a portion of the profit it invests in increasing its capacity $g^i_m = \lambda(p^i_m - c^i_m) = \frac{\dot{y}^i_m}{y^i_m}$. Unless specified differently, we keep parameter $\lambda = 0.01$;
- in the first part of the model firms' productivity is fixed, while in the later parts we relax this assumption so that firms investing in innovation (out from positive sales) can improve own performance.

Since our primary unit of interest is unit cost per value added, in the following we can compare it to the standard unit cost of production C^i_m:
for the first layer of value chain (layer 1) the two are assumed to be identical $C^i_1 = c^i_1$ as the cost of adding value can be refered to the cost of extracting primary resources of production;
for the second layer of the value chain, $C^j_2 = c^j_2 + p^j_1 = c^j_2 + c^j_1(1+\phi)$; ...
for the Mth layer of production, $C^j_M = c^j_M + p^j_{M-1} = c^j_M + \sum_{m=1}^{M-1} c^j_m(1+\phi)$.

From the replicator model we know that market share $s^i_m = \frac{y^i_m}{y_m}$ of firm i on market m changes according to the following selection equation:

$$\dot{s}^i = \frac{\partial s^i}{\partial t} = s^i \lambda(\bar{c} - c^i) = \frac{\dot{y}^i y - y^i \dot{y}}{y^2}. \tag{1}$$

Now, given that we consider simplistic value chain structure with one element in each layer only and no other markets (or firms) connected to those considered here, one can reasonably argue that the unit output of firm j in the final layer must be equal to its supplier's one in each preceding layer $y^i_M = y^i_{M-1} = ... = y^i_1$, while the total unit output of market M - to the preceding ones: $y_M = y_{M-1} = ... = y_1$. As a consequence, the following equalities must hold:

$$\dot{y}_M = \dot{y}_{M-1} = ... = \dot{y}_1 \tag{2}$$

$$\dot{y}^j_M = \dot{y}^j_{M-1} = ... = \dot{y}^j_1 \tag{3}$$

and consequentially

$$s_M = s_{M-1} = ... = s_1 \tag{4}$$

$$\dot{s}^j_M = \dot{s}^j_{M-1} = ... = \dot{s}^j_1. \tag{5}$$

4 Simulation Results

In the following we list exercises addressing them (whenever suitable) analytically and computationally.

Exercise A. Consider two contrast scenarios: in the first one firms located on each market m have their productivity (measured in unit cost per value added) drawn in a way that each firm surpasses the next one by the same amount (e.g., 1, 1.5, 2,...), but the firms integrated in a value chain are matched according to their productivity: most fit firm in market M with most fit ones in market $M-1$, $M-2$ etc and the other way around. In the opposite scenario, firms having their fitness drawn the same way are matched randomly - some less fit firms may be matched with more fit ones.

For that, let us denote most fit firm in each layer with index a, second most fit firm with index b and (for the simplified case of three firms only) the least fit firm with index c. Hence, in the ordered matching we have all a firms linked together (having aggregate unit cost C^a_M), while in the random matching - they are randomly distributed in different value chains (VCs). Therefore, in the ordered matching scenario (our benchmark) the most fit firm in each layer increases its market share similar to equation 1. In particular,

$$\dot{s}^a_M = \frac{\partial s^a}{\partial t} = s^a_M \lambda(\bar{C} - C^a_M). \tag{6}$$

The difference between (6) and (1) is that the 'monopolization' takes place for the VC case even faster since

$$\begin{aligned} C^a_M - C^b_M =& c^a_M + \sum_{m=1}^{M-1} c^a_m (1+\phi) - c^b_M - \sum_{m=1}^{M-1} c^b_m (1+\phi) = \\ =& c^a_M - c^b_M + (1+\phi)\left(\sum_{m=1}^{M-1} c^a_m - \sum_{m=1}^{M-1} c^b_m\right) > c^a_M - c^b_M. \end{aligned} \tag{7}$$

In the random matching scenario, in contrast, the monopolization takes place potentially much slower since in each layer firms with different fitness are matched. Eventually, one of the value chains certainly dominates the other one (as long as its aggregate fitness is lower), but this has a (negative) side effect in a sense of a less fit firm in one (or more than one layer) dominating with its market share its counter-partners. To illustrate that, consider Figure 1. The leftmost charts in the upper and lower panels display the differences in the speed of market reallocation; the mid charts show the corresponding dynamics with respect to change in the (aggregate) average unit costs $\bar{C}_M$;[4] Finally, the rightmost charts in Figure 1 show that while in the ordered matching the best firm dominates the market in *each* layer driving the average unit cost per value added to its lowest value which is one), in the random matching - that is far not necessarily the case. Thus, in the specific example illustrated the least efficient firm in the layer two dominates the market, which can be seen from the average unit cost per value added in this layer converging to the value of two, which is the lowest efficiency present on that market.

[4]Note that so far no innovation (in the sense of autonomous cost improvements by firms) is allowed and only the market share reallocation dynamics is driving the aggregate productivity of the model.

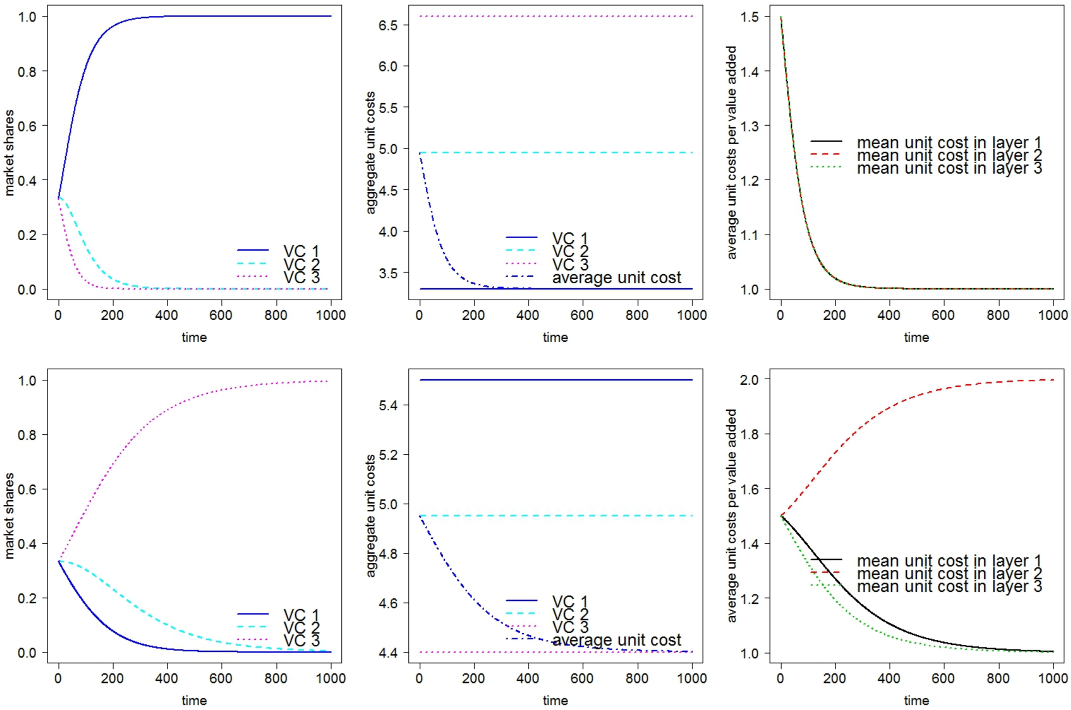

Figure 1: Replicator dynamics in value chain with ordered and random matching

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$ and $N = 3$.

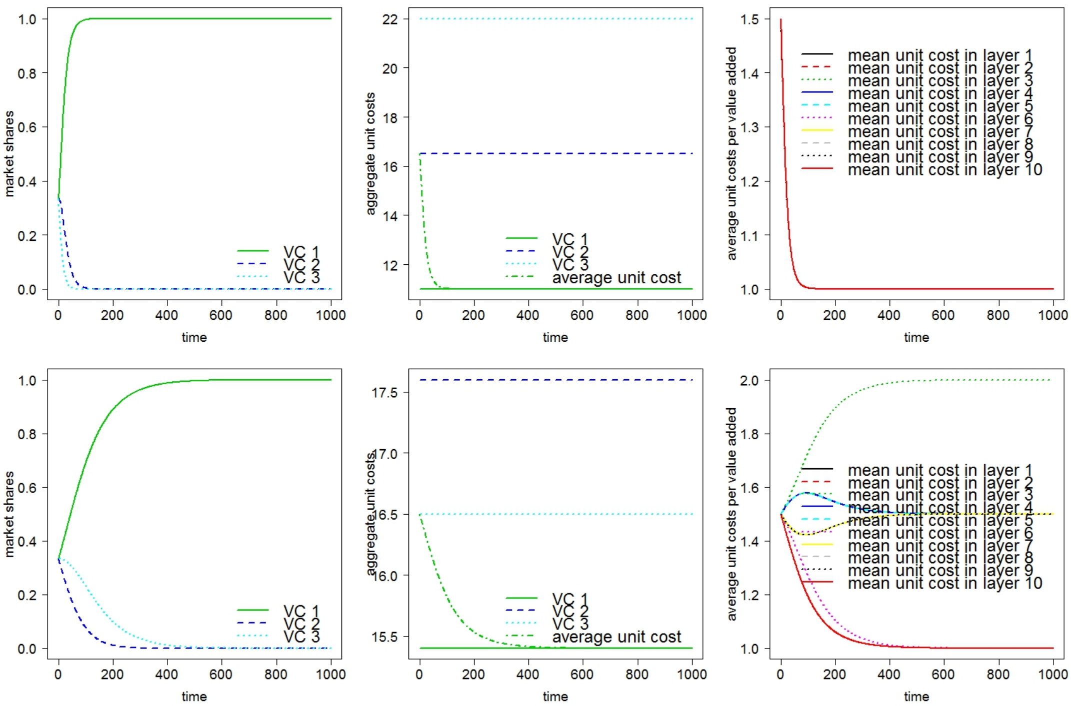

Figure 2: Replicator dynamics with ordered and random matching with ten layers

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 10$ and $N = 3$.

Analytically this can be supported in the following way. Remember that $\bar{c}_2 = \sum_{n=1}^{N} s_n C_n$ and given that in our particular case (through a random event) the least fit firm from the second layer has been matched with most fit firms in layers one and three, the aggregate fitness of that value chain (VC3 on the bottom mid chart of Figure 1) is lowest, and hence, it is merely a question of time when this value chain and effectively least fit firm in layer two will dominate the market (see equation (6)).

If one increases the number of layers M from three to, e.g., ten, then the difference in aggregate fitness between the value chains measured by aggregate unit costs C_M^j will increase and domination of one value chain over other competitor chains will take place faster (Figures 2).[5] As a result, the gain in speed of market reallocation is most evident for ordered matching, where the term $\left(\sum_{m=1}^{M-1} c_m^a - \sum_{m=1}^{M-1} c_m^b\right)$ is increasing with every new layer of a value chain. For random matching scenario, the contribution of a larger M is not evident since the value chains are matched randomly and on average shall contain for different M the same portion of more (a) or less (c) fit agents:

$$E[C_M] = c_M^a * \frac{1}{N} + c_M^b * \frac{1}{N} + c_M^c * \frac{1}{N} + \sum_{m=1}^{M-1} \left(c_m^a * \frac{1}{N} + c_m^b * \frac{1}{N} + c_m^c * \frac{1}{N} \right) (1 + \phi). \tag{8}$$

Therefore, any difference in speed of market reallocation between bottom leftmost charts in Figures 1-2 is due to a random event (particular random VC matching) and not to any objective criterion.

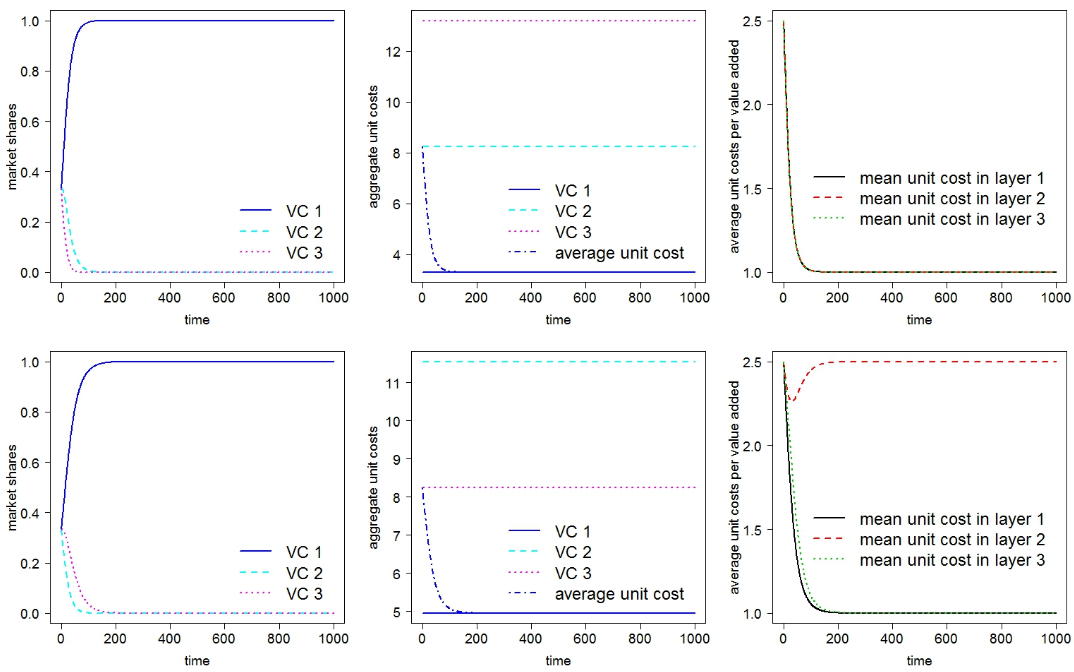

Figure 3: Replicator dynamics with ordered and random matching with larger variance in fitness

Note: While in the default case, as it was mentioned earlier, firms' productivity has been drawn in a way that each firm surpasses the next one by 0.5 (which was leading to $(\sigma_c^m)^2 \approx 0.167$), here we increase the step to 1 and, respectively, the$(\sigma_c^m)^2$ to ≈ 0.67.
The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 10$ and $N = 3$.

[5]Note here that we keep the variance in unit costs per value added fixed in each layer, and increasing M naturally leads to larger differences between value chains.

Furthermore, increasing the variance in fitness between firms on each layer $(\sigma_c^m)^2 = \sum_{n=1}^{N} (c_m^n - \bar{c})$ in our model, one increases the differences in expected aggregate unit costs between the value chains, which automatically leads to faster market reallocation process. This result holds for both, ordered and random matching scenarios. While a careful derivation of that result still has to be done, a parallel with the classical *Fisher's theorem* stating that the change in average fitness in a population of competing firms is proportional to the variance in fitness is obvious and is illustrated in Figure 3.

What one can also learn from (7-8) is that the speed of market reallocation is additionally boosted by profit margin ϕ since it increases the variance in unit cost in the final layer M similarly affecting the ordered and random matching scenarios (se Figure 4).

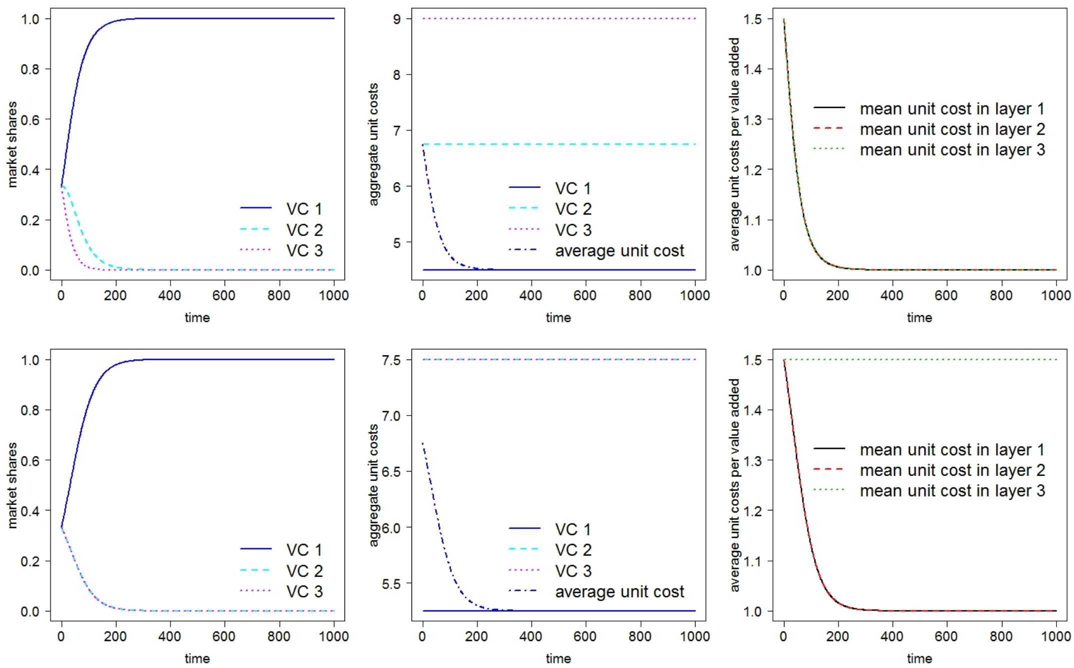

Figure 4: Replicator dynamics with ordered and random matching with alternative profit margin

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 10$, $N = 3$ and $\phi = 0.5$.

Hence, based on the Exercise A the following hypotheses can be formulated:

Hypothesis 1 *Assuming firms having different fitness, being vertically integrated in value chains, and imperfectly matched with each other in terms of their performance, one would expect ceteris paribus a slower market selection process than if no vertical integration would have been present.*

Hypothesis 2 *A larger variance in fitness between firms on each layer increases proportionally the speed of market reallocation both, for ordered and random matching scenarios. A similar effect has a larger profit margin demanded by firms in each layer ϕ.*

Hypothesis 3 *A larger number of value chain layers M increases the speed of market reallocation, but only in case of value chains matched according to the performance rank of its members (ordered matching scenario).*

One can notice that Hypothesis 3 is an extension Hypothesis 2, as M contributes to the variance in aggregate fitness of value chains, but only for ordered matching.

Hypothesis 4 *A firm with a fitness below average of the market it is operating in may dominate it if it is integrated with highly fit partners from other layers making the overall fitness of the value chain highest on the final end consumer market (layer M).*

Exercise B. Take the previous exercise but allow firms to invest in R&D from their positive profits. A result of R&D should be a continuously increasing productivity of firms similar to Dasgupta and Stiglitz (1980). As a result, firms with lower fitness can catch-up because of their integration with strong partners and the other way around.

To illustrate that, we adopt few alternative specifications of innovation process: with constant (Figure 5), decreasing (Figure 6) and increasing (Figure 7) returns. Following Mazzucatto (1998), this is done by setting

$$c_m^j(t+1) = c_m^j(t)\,(1-\gamma) \qquad \text{for constant returns} \tag{9}$$

$$c_m^j(t+1) = c_m^j(t)\,\left(1-\gamma(1-s_m^j(t))\right) \qquad \text{for decreasing returns} \tag{10}$$

$$c_m^j(t+1) = c_m^j(t)\,\left(1-\gamma s_m^j(t)\right) \qquad \text{for increasing returns.} \tag{11}$$

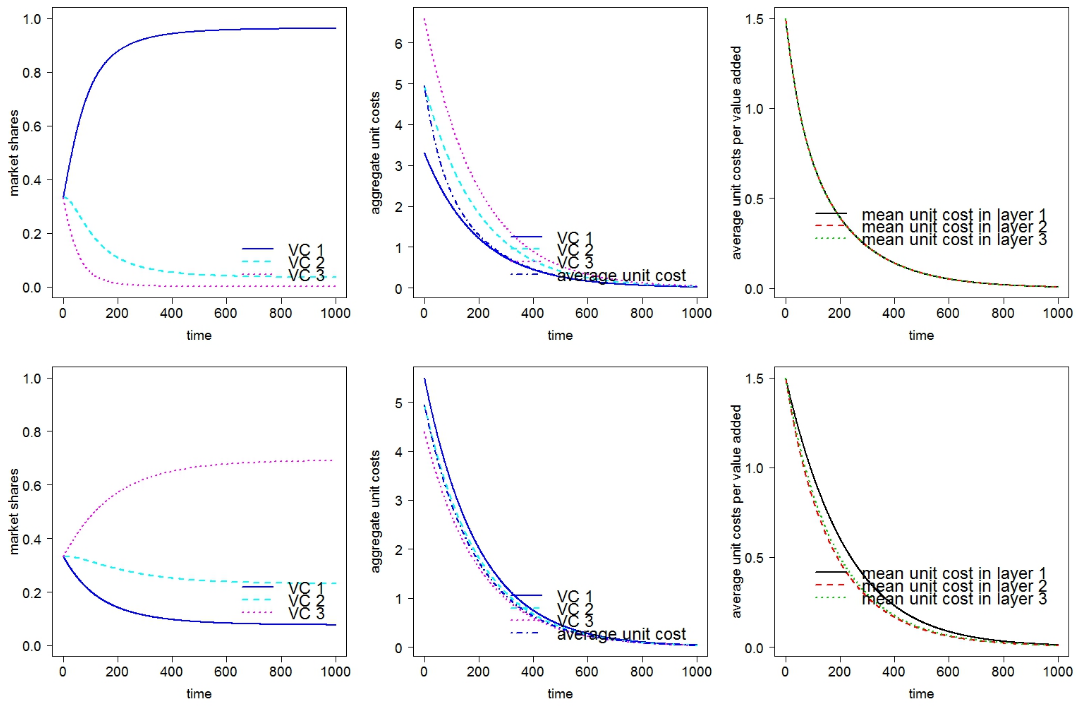

Figure 5: Replicator dynamics with ordered and random matching and innovation with constant returns to scale

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$ and $\gamma = 0.005$.

The possibility of cost reduction with constant returns to scale, as expected in accordance with the standard replicator model, creates the possibility of more than one value chain staying on the market (see leftmost charts in Figure 5). Since in the ordered matching, the difference in total unit costs between the value chains is originally larger, the dominating value chain achieves a higher market share than in the case of random matching. The fact that the less fit firm obtains an advantage through integration with strong partners in other layers can also be seen from Figure 5. For that, note first that the aggregate unit costs at the starting point in the random matching are much more similar than in the ordered one indicating the fact of random matching. Then in the rightmost charts one notices that while the progress of average unit costs per value added in each layer is uniform for ordered matching, there are some differences in the random matching with layer one lagging clearly behind the other two. The only explanation for that[6] is the fact that a firm integrated in the winning VC from that layer had originally lower fitness and it took it more time to improve it.

For decreasing returns to scale: Setting the rate of innovative change to be inversely proportional to market share, one obtains a typical pattern of high volatility of market shares in the initial period. This volatility is (potentially) higher in the ordered matching, where the differences in fitness between the value chains are higher.

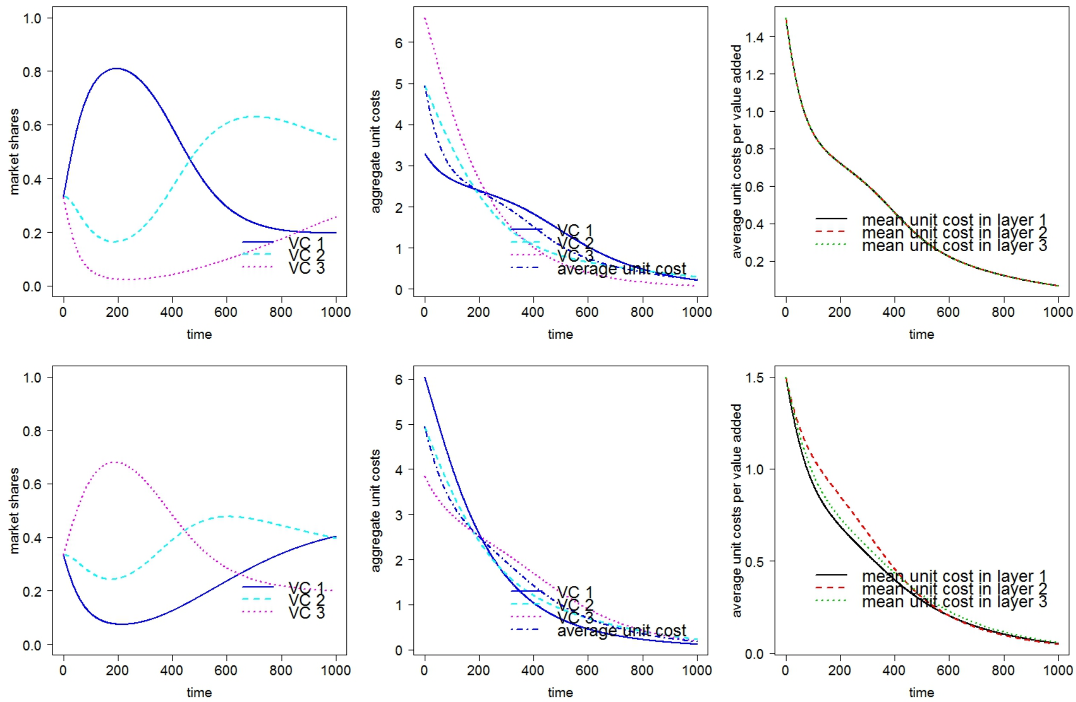

Figure 6: Replicator dynamics with ordered and random matching and innovation with decreasing returns to scale

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$ and $\gamma = 0.005$.

For increasing returns to scale: As it is typical for increasing returns to scale, a random event (in terms of slightly lower unit cost at the beginning of simulation) defines which of the value chains will dominate the others. Once firms start innovating, evolution of unit costs and market shares (at least for the leading value chain) proceeds much faster than in the scenario with constant returns to scale (Figure 7). The process of market

[6]This is since market shares of all three firms integrated in the winning VC rise, as it was stated earlier, at the same rate.

monopolization is taking place again faster in ordered matching as the initial advantage of the fittest value chain over its counterparts is larger.

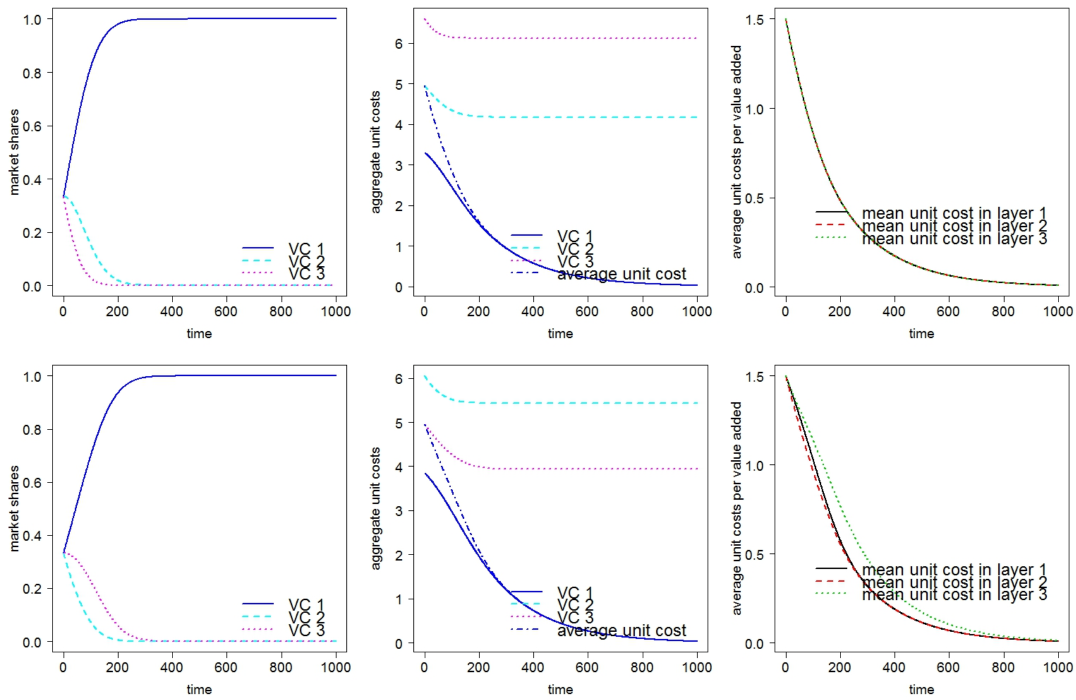

Figure 7: Replicator dynamics in value chain with random matching and innovation with increasing returns to scale

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$ and $\gamma = 0.005$.

Hypothesis 5 *The possibility of innovation with constant returns to scale in the random matching scenario creates more opportunities for more than one actor (value chain) staying on the market than in the ordered matching scenario.*

Hypothesis 6 *Considering the cases of decreasing and increasing returns to scale and applying them to vertically integrated value chains, one identifies a similar pattern of fitness evolution and market concentration with the difference that the typical market share volatility and fast market share concentration (respectively) are pronounced for the ordered matching scenario.*

Hypothesis 7 *Extending hypothesis 4, one can argue that a less fit firm integrated in a superior value chain gets an opportunity to improve its fitness once innovation process is allowed. This, however, is true only for constant or increasing returns to scale, while it is the opposite for decreasing returns to scale.*

Exercise C. While in scenarios A-B the value chains were assumed to be fixed due to prohibitively high switching cost, one could relax that assumption. Switching cost may involve simply a fixed cost FC (in case of homogenous products in intermediate layers), and those firms which either compensate this cost by gaining lower price of a new supplier multiplied by existing orders and/or gaining more orders requested by new downstream partner, will be willing to switch.

To account for the fact that a firm can switch only if there is reciprocity from the other side (potential partner finds it also attractive to switch to that firm), we introduce a simple search and acceptance algorithm ensuring reciprocity. In particular, if a firm j from a layer m considers to switch its current partner jj from a layer mm (which can be either $m+1$ or $m-1$) and takes (randomly) firm $jk \neq jj$ into consideration (which in its turn has currently a partnership with firm kk from layer m), then those two firms, j and jk, will do the switching *iff*:

$$c_{mm}^{jj} - c_{mm}^{jk} > FC \tag{12}$$

$$c_{m}^{kk} - c_{m}^{j} > FC. \tag{13}$$

Necessarily, the parameter of switching cost $FC \in [0, \infty)$ becomes a key parameter in that respect allowing situations from 'fast and easy' switching for the two firms as if no sunk costs of partnership formation exist to no switching at all.

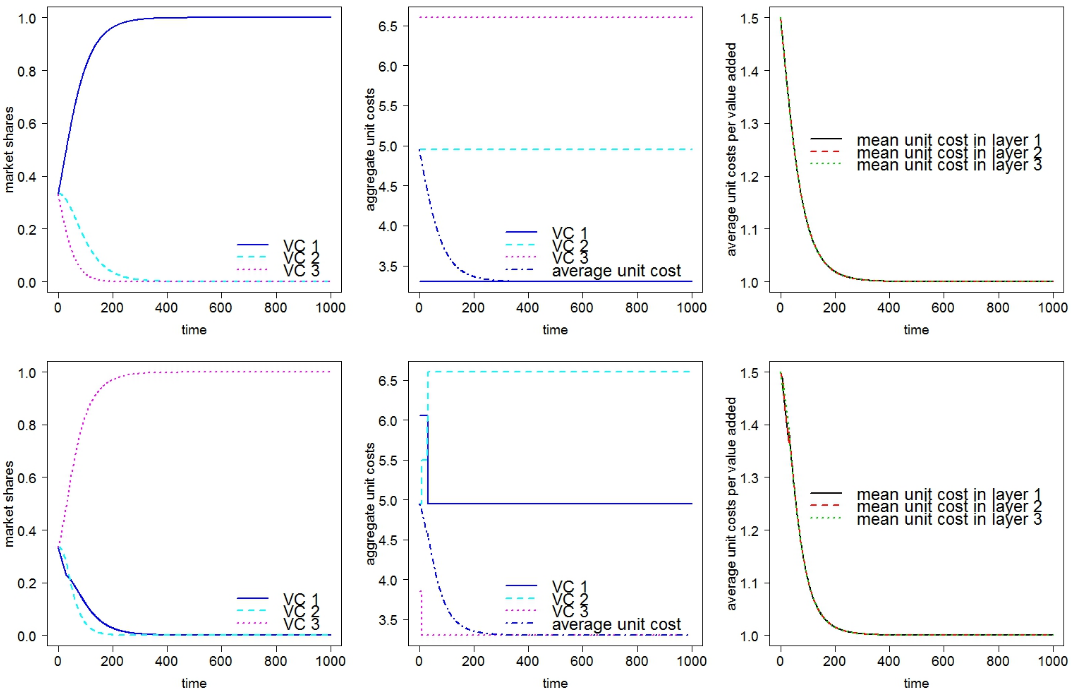

Figure 8: Replicator dynamics in value chain with ordered and random matching, no innovation and switching

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$, $FC = 0.001$ and $\gamma = 0.005$.

As in the ordered switching scenario fittest firms in the respective layers are matched together, there is basically no room for switching left (upper panels in Figures 8-11. In contrast, in case of random value chain matching, firms always switch (no matter whether innovative activity is present and if yes, in which scenario of scale returns). The moment of switching can be captured by 'zig-zag' evolution (abrupt shifts) of the aggregate unit costs of the VCs (mid-charts in the lower panels of Figures 8-11).

As a result, in early periods of simulation (which can be interpreted as early stage of industry life-cycle) one observes a period of volatility in market share constellation. This is particularly pronounced in cases of decreasing but also increasing returns to scale. In the latter case it happens because weaker firms being integrated in originally superior value chain and having improved their performance may find attractive to switch later their partner to an even stronger one from a different VC, thus, creating an even more superior actor (value chain). This finding contrasts the earlier argument made by Mazzucato (1998) that high volatility in the early period of life-cycle happens only for the decreasing returns scenario demonstrating that the volatility is much more universal and not so sensitive to scale returns.

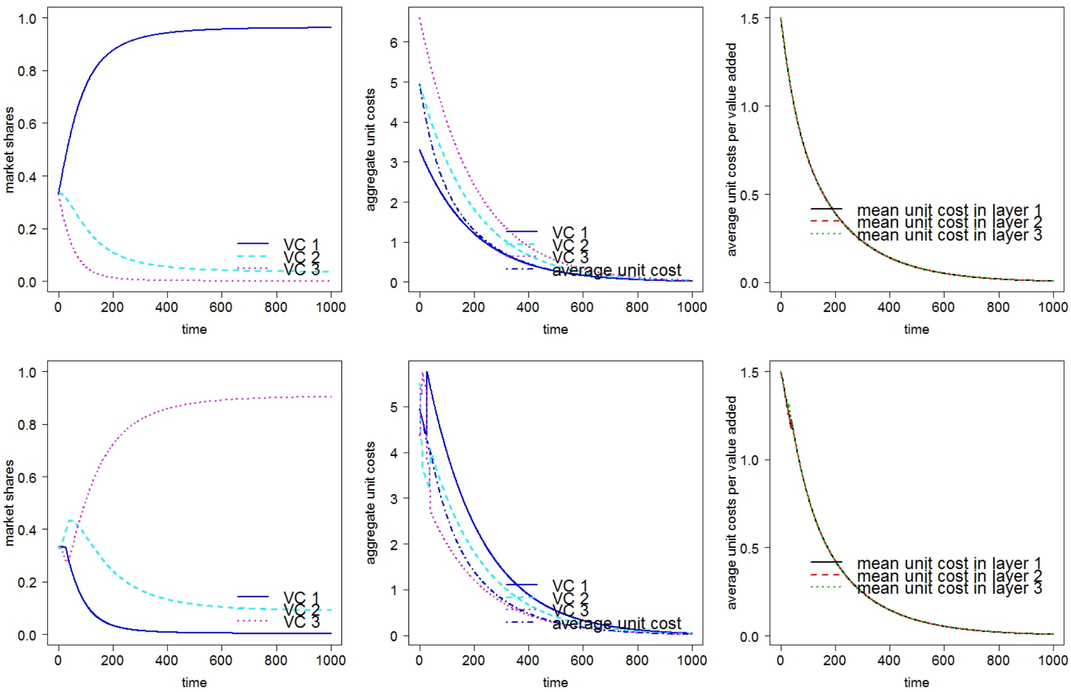

Figure 9: Replicator dynamics in value chain with ordered and random matching, constant returns to scale and switching

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$, $FC = 0.001$ and $\gamma = 0.005$.

Another pronounced effect from switching is that the random matching scenario gains in speed of industry monopolization. Clearly, setting FC low, one allows firms to quickly rearrange their link constellation so that fittest firm will be integrated with fittest partners in other layers. As a result, progress in aggregate productivity and market monopolization take place faster with FC determining time delay in this evolution: from very small to infinite.

Hypothesis 8 *Considering random matching scenario of vertically integrated firms and allowing them to switch, one can observe high volatility in market share dynamics at the beginning of simulation (corresponding to early period of industry life-cycle) irrespective of the specific return to scale case.*

Hypothesis 9 *Possibility of switching fosters dynamics in market share reallocation and productivity improvement. Hence, the smaller the switching cost FC, the more efficient the markets.*

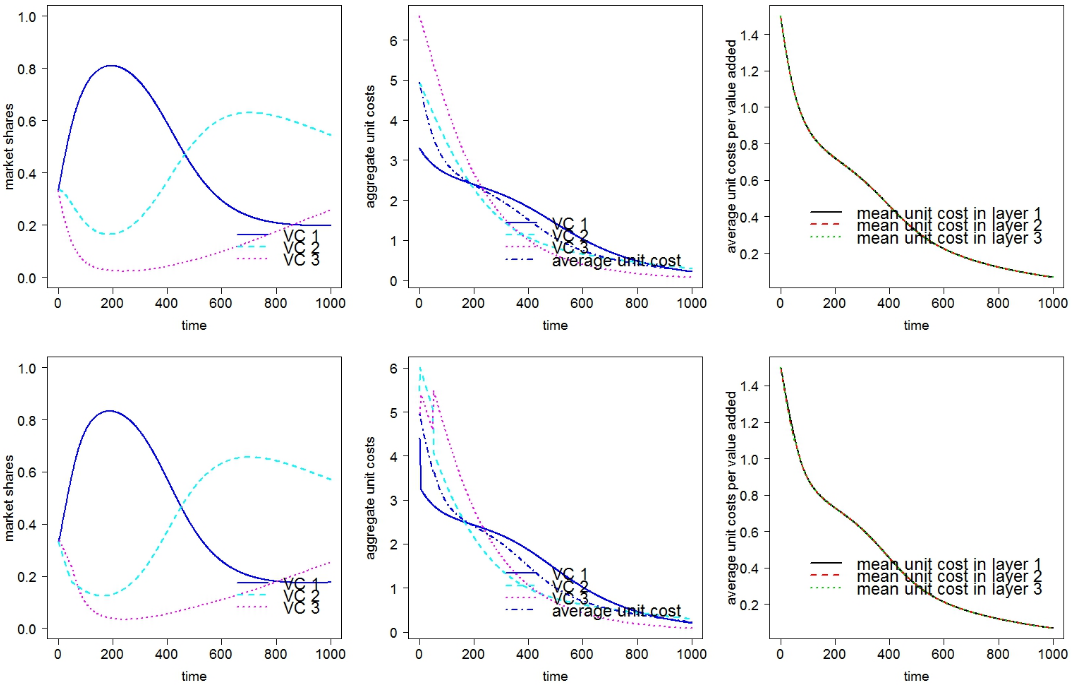

Figure 10: Replicator dynamics in value chain with ordered and random matching, decreasing returns to scale and switching

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$, $FC = 0.001$ and $\gamma = 0.005$.

The latter hypothesis 9 has a considerable potential for policy implication. Although policy maker has generally limited influence on firms' startegic decision with regard to partner selection, certain measures such as increasing market transparency or financial support for firms at the early period of alliance formation may come in question.

5 Discussion and Conclusion

In this paper we make an attempt to generalize the replicator model to the case of vertically integrated firms. For that we conduct a series of exercises starting from most simplest one where no innovation of partner switching is allowed and increasing the complexity stepwise. Doing this, we contrast two scenarios with firms being matched according to the performance rank of its members (ordered matching scenario) and those being matched completely randomly. Then, using some analytical but mainly computational tools, we show how the two scenarios differ and address the differences also to the standard replicator model without any market integration.

A result of this exercises is a series of hypotheses 1-9, which can be further addressed empirically. Below we also address some challenges related to their empirical testing.

For further research we plane to generalize the exercise allowing firms to partner more than one firm from the same layer at the same time. This should allow to address network properties resulting from the exercise. Furthermore, one can be able to draw some intuition on differences between layers in terms of their alliance formation power and firm survival. All these are challenging and important questions we hope to address soonest possible.

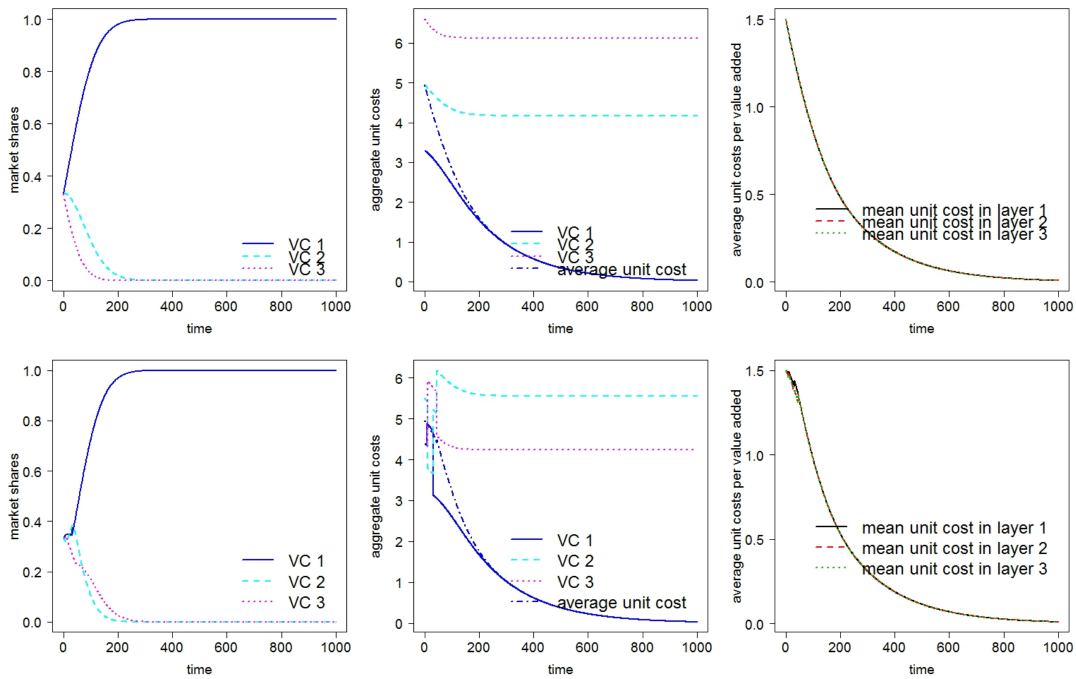

Figure 11: Replicator dynamics in value chain with ordered and random matching, increasing returns to scale and switching

Note: The upper panel corresponds to ordered matching, while the lower - to random matching. $M = 3$, $N = 3$, $FC = 0.001$ and $\gamma = 0.005$.

Outlook for empirical analysis:

The major claim (hypothesis) to be tested empirically is that the degree of vertical integration of an industry (assuming that all firms present in the industry (market) have the same degree of integration) changes the speed of market reallocation (most likely reducing it as one expects that firms are not perfectly matched within those value chains, see Exercise A). The challenge is than to measure the degree of vertical integration at industry (or ideally at firm) level and use this as a variable in explaining the share of marginally growing firms (similar to the definition in Hölzl (2015)).[7] One source of data to measure the degree of vertical integration could be the money flows between industries released with input/output OECD tables (McNerney et al., 2013) (which data is largely aggregated to UN standardization between national accounting systems) or so called use tables illustrating expenditure of each industry on individual commodity and achieving by this a more disaggregated level of information (Carvalho, 2010).

In addition further explanatory variables such as extent of mobility barriers, degree of product differentiation and other controls should be taken into account. In case the degree of vertical integration being identified negative and significant, one could not reject the hypothesis on the role of value chain integration in reducing market share reallocation.

A more sophisticated but also more reliable way for empirical testing would require to assess fitness of all firms integrated within a value chain and, having a sufficient number of cases (chains), test whether firms integrated with less fit partners where experiencing a negative effect on their performance and the other way around. This , however, has an even more challenging data collection process.

[7]Ideally, the firms or industries with high degree of vertical integrated should be tested against firms/industries performing the complete production cycle (from raw material to finished goods) alone.

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